



Bi-level Programming Based Optimal Strategy to LSEs with Demand Response Bids

Ding, Tao; Yang, Qingrun; Jiang, Jiangfeng; Yang, Yongheng; Blåbjerg, Frede; Li, Yaohong; Ju, Yuntao

Published in:
Electric Power Components & Systems

DOI (link to publication from Publisher):
[10.1080/15325008.2018.1528315](https://doi.org/10.1080/15325008.2018.1528315)

Publication date:
2018

Document Version
Accepted author manuscript, peer reviewed version

[Link to publication from Aalborg University](#)

Citation for published version (APA):
Ding, T., Yang, Q., Jiang, J., Yang, Y., Blåbjerg, F., Li, Y., & Ju, Y. (2018). Bi-level Programming Based Optimal Strategy to LSEs with Demand Response Bids. *Electric Power Components & Systems*, 46(18), 1926-1937. <https://doi.org/10.1080/15325008.2018.1528315>

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal -

Take down policy

If you believe that this document breaches copyright please contact us at vbn@aub.aau.dk providing details, and we will remove access to the work immediately and investigate your claim.

RESEARCH ARTICLE

Bi-level Programming Based Optimal Strategy to LSEs with Demand Response Bids

Tao Ding¹ | Qingrun Yang¹ | Jiangfeng Jiang¹ | Yongheng Yang² | Frede Blaabjerg² | Yaohong Li³ | Yuntao Ju⁴

1. State Key Laboratory of Electrical Insulation and Power Equipment, School of Electrical Engineering, Xi'an Jiaotong University, Xi'an, 710049, China;

2. Department of Energy Technology, Aalborg University, Pontoppidanstraede 101, Aalborg 9220, Denmark

3. State Grid Jiangsu Electric Power Company, Nanjing 210024, China

4. Department of Electrical Engineering, China Agricultural University, Beijing 100083, China

Summary

With the increasing demand-side participation in electricity market, as a profit-seeking market participant, load-serving entities (LSEs) have been trying to apply demand response (DR) programs to induce the demand elasticity to further their profit. However, due to the different preference of DRs, it is difficult for LSEs to generate the optimal strategic bidding strategy considering DR in the ISO/RTO's market.

Therefore, this paper proposed a bi-level optimization model with the consideration of demand response bidding to maximize the total profit of LSEs: 1) conceptually, different from previous related works, the consumers participate DR through setting their bidding prices to LSEs with respect to their own preference and LSEs should determine the optimal reward value of DR as well as the amount of demanded electricity; and 2) technically, an original method has been implemented to solve the bi-level optimization model. The closed form of shadow price function with respect to the total load demand is derived to reduce the complexity of the proposed bi-level model. Hence, the proposed model is converted to a mixed integer second order cone programming and able to achieve the global optimality. It needs to be note that the closed form of shadow price introduced in this paper can also be applied to other bi-level programming models. Moreover, case studies have been performed to demonstrate the validity of the proposed method: 1) the proposed method to obtain the closed form of real-time price is verified on a 9-bus system; 2) 118-bus test system with three demand response participants is tested to show that by the proposed method, LSE can benefit from the DRs under various circumstance.

KEYWORDS

Demand response, mixed integer second order cone programming, optimal strategy, bi-level programming, load serving entities (LSEs)

1. Introduction

With the increasing demand-side participation in the electricity market, electricity providers, such as load serving entities (LSEs), have been offering incentives to the businesses to adjust their energy usage pattern to

improve the system efficiency and maximize the operating profit by reducing the peak or occasional demand spikes [1]-[3]. Generally, there are two types of demand response (DR) programs that have been widely adopted by LSEs: incentive based program (IBP) and price-based program (PBP) [4]. As for IBP, it can be divided into several typical

types including direct load control [5]-[6], interruptible program [7], market based IBP including emergency demand response [8], demand bidding [9]-[11], capacity market [12] and ancillary services market [13]. As for PBP, the various pricing mechanisms have been implemented including time-of-use rate (TOU) [14], critical peak pricing (CPP) [15], peak load pricing (PLP) [16] and real-time pricing (RTP) [17-20]. In addition, a coupon based demand response was proposed in [21] to optimize the coupon price.

As a profit-seeking market participant, LSEs purchase electricity with real-time prices from wholesale market and charge consumers with flat rate. Therefore, LSEs are taking the risk of financial loss whenever the real-time price in whole market is higher than retail flat rate. To hedge against the risk, several measures have been adopted in electricity markets, such as contracts, futures and etc.. Recently, demand response acts as another way that has been applied by LSEs to induce the inherent elasticity of the demand to moderate the peak demand. However, the difficulty for implementing DR is to accurately model its uncertainty. In practical, DR is diverse with respect to the types of consumers and varies with different time periods.

There have been considerable amount of works trying to address the uncertainty in DR: [22] proposed a multi-stage robust optimization method to find an optimal solution under the worst case scenario; [23] introduced index policies for DR considering unknown demand capability; [24] adopted uncertain optimization decision of interruptible load to study the uncertain customer response and total interruptible capacity requirement; [25] studied a stochastic unit commitment with the consideration of demand response uncertainty.

Intuitively, consumers themselves could comprehensively evaluate the loss when they participate the demand response programs, and well address the accurate demand response characteristics. Hence, different from the existing methods, this paper considers the DR mechanism, in which all DR participants send their bidding curve and capacity to LSE, according to their preferences, lifestyles, and etc. Then LSE determines the optimal reward value of DR as well as the amount of demanded electricity in performing the market simulation to maximize their profits.

In the practical electricity market, this kind of program is usually participated by big industrial energy consumers, such as supermarkets, buildings, companies and so on. Certainly, for each individual energy user, load aggregation can be utilized through which individual energy users are banded together in an alliance to more competitive prices than they might otherwise receive working independently [26]-[28]. Aggregation can be accomplished through a simple pooling arrangement or through the formation of clusters where individual contracts are negotiated between the suppliers and each member of the aggregate group.

The schematic of the market structure with such DR mechanism is as Fig. 1. Under this framework, the optimal strategy model for LSE becomes a bi-level model, where the inner model is the economic dispatch model to determine the real-time price and the outer model is to maximize the total profit of LSE.

Technically, the closed form of shadow price function with respect to the total load demand is derived to reduce the complexity of the proposed bi-level model. Therefore, the proposed model is converted to a mixed integer second order cone programming and able to achieve the global optimality. It needs to be note that the closed form of shadow price introduced in this paper can also be applied to other bi-level programming models.

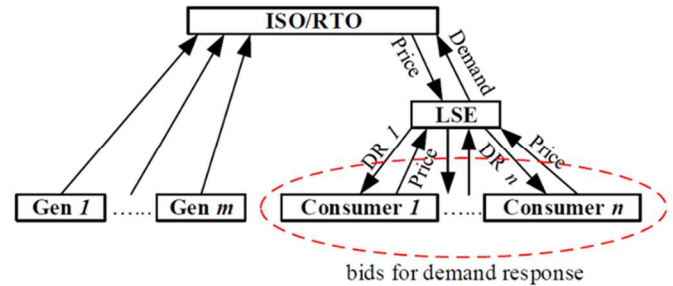


Fig. 1. Demand response bidding for LSE

The rest of the paper is organized as follows: Section II presents the procedures of LSEs' operation with the consideration of bidding based DR, and further formulates this issue as a bi-level linear optimization model. Then, in order to solve this model, the closed form of inner model is introduced in Section III to convert the model from bi-level into single level. In Section IV, mixed integer conic programming is utilized to achieve to the global optimum of the original model. Case studies on a 118-bus system with three demand response participants are shown in Section V with comparisons between the profit of LSE with and without DR, with respect to various retail prices. Finally, conclusions are drawn in Section VI.

2. LSE's Optimal Strategy Considering DR

For each time period, ISOs' economic dispatch (ED) aims to determine the optimal power generation of each dispatchable unit and minimize the total operating costs of serving the system's demand. The problem formulation of ED is derived as a convex quadratic programming problem as follows:

$$\text{ED) min } \sum_{i=1}^{N_g} (a_i P_i^2 + b_i P_i + c_i) \quad (1-a)$$

$$\text{s.t. } \sum_{i=1}^{N_g} P_i = \bar{D} \quad (1-b)$$

$$P_i^{\min} \leq P_i \leq P_i^{\max}, \quad i = 1, \dots, N_g \quad (1-c)$$

where (a_i, b_i, c_i) is the triplet fuel cost function of unit i ; N_g denotes the number of dispatchable generators; P_i is the

power output of dispatchable unit i ; \bar{D} is the total demand without DR; minimum/maximum generation capacity of unit i are P_i^{\min}/P_i^{\max} ; constraints (1-b) and (1-c) represent the constraints of energy balance and generator capacity respectively.

While the actual models in practice are more complex, the simplified ED model without considering transmission capacity constraints, as in [21], is utilized here to illustrate the main point of the proposed work. Hence, the difficulty of obtaining the ISO's exact network information for the market participants is avoided. It should be noted that although the discussion in this paper ignores the transmission capacity constraints, the simple ED model can still provide valuable potential insights on DR to those market participants as well as facilitate the bi-level DR modeling.

As a market participant, the objective for LSE is maximizing the total operating profit. Since the DR offers LSEs the opportunities to induce the demand elasticity to increase the profit, the modified optimization problem considering the DR from N_r consumers can be expressed as below:

$$(DR) \quad Z = \max \quad (\lambda_0 - \lambda) D - \sum_{r=1}^{N_r} \pi_r D_r \quad (2-a)$$

$$s.t. \quad 0 \leq D_r \leq D_r^{\max}, \quad r = 1, \dots, N_r \quad (2-b)$$

$$D = \bar{D} - \sum_{r=1}^{N_r} D_r \quad (2-c)$$

$$\lambda = \arg \min_{P_i} \sum_{i=1}^{N_g} (a_i P_i^2 + b_i P_i + c_i) \quad (2-d)$$

$$s.t. \quad \sum_{i=1}^{N_g} P_i = D: \quad \lambda \quad (2-e)$$

$$P_i^{\min} \leq P_i \leq P_i^{\max}, \quad i = 1, \dots, N_g \quad (2-f)$$

where λ_0 and λ are electricity retail and real-time prices; D is the actual demand with DR; π_r is the bidding price of consumer r in DR; D_r^{\max} is the maximum load shedding for DR; N_r is the number of the consumers in DR programs. Constraint (2-b) is the upper and lower bound for load demand response; constraint (2-c) describes the true load demand after the load demand response; constraints (2-d)-(2-f) are the inner ISOs' economic dispatch model constraining the real-time price λ .

Based on the common sense of market economics, a consumer's bidding price should be increasing with the incremental of his/her demand. Here, in this paper, the bidding price for an electricity consumer is modeled as a piecewise curve as presented in Fig. 2.

Hence, the consumers' bidding prices can be formulated as:

$$\pi_r = \begin{cases} \pi_{r,1r} & \text{if } 0 \leq D_r \leq D_{r,1} \\ \pi_{r,2} & \text{if } D_{r,1} \leq D_r \leq D_{r,2} \\ \dots & \dots \\ \pi_{r,m_r} & \text{if } D_{r,m_r-1} \leq D_r \leq D_r^{\max} \end{cases} \quad (3)$$

where $\pi_{r,1}, \pi_{r,2}, \dots, \pi_{r,m_r}$ are the segments of the bidding prices for consumer r ; $D_{r,1}, D_{r,2}, \dots, D_{r,m_r-1}$ are the demand segments of the bidding curve for consumer r . Then, the total electricity costs for consumer r should be:

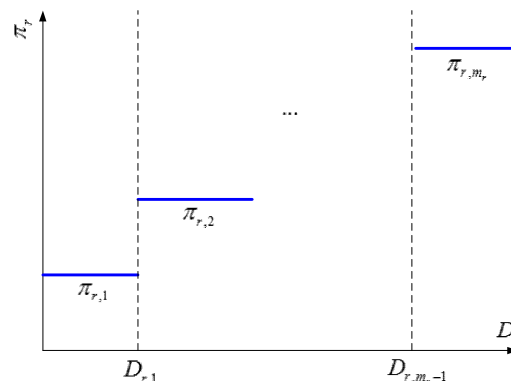


Fig.2 Bidding price curve for consumer r

$$\pi_r D_r = \begin{cases} \pi_{r,1} D_r & \text{if } 0 \leq D_r \leq D_{r,1} \\ \pi_{r,1} D_{r,1} + \pi_{r,2} (D_r - D_{r,1}) & \text{if } D_{r,1} \leq D_r \leq D_{r,2} \\ \dots & \dots \\ \pi_{r,m_r-1} D_{r,m_r-1} + \pi_{r,m_r} (D_r - D_{r,m_r-1}) & \text{if } D_{r,m_r-1} \leq D_r \leq D_r^{\max} \end{cases} \quad (4)$$

It can be observed that the function $\pi_r D_r$ is a convex function that can be reformulated as:

$$\pi_r D_r = \max \left\{ \pi_{r,1} D_r, \dots, \pi_{r,m_r-1} D_{r,m_r-1} + \pi_{r,m_r} (D_r - D_{r,m_r-1}) \right\} \quad (5)$$

Furthermore, an additional variable s_r is employed here to simplify the model (2) into (6).

$$(DR1) \quad Z = \max \quad \lambda_0 D - \lambda D - \sum_{r=1}^{N_r} s_r \quad (6-a)$$

$$s.t. \quad 0 \leq D_r \leq D_r^{\max}, \quad r = 1, \dots, N_r \quad (6-b)$$

$$D = \bar{D} - \sum_{r=1}^{N_r} D_r \quad (6-c)$$

$$\begin{cases} \pi_{r,1} P_r \leq s_r \\ \pi_{r,1} P_{r,1} + \pi_{r,2} (P_r - P_{r,1}) \leq s_r \\ \dots \\ \pi_{r,m_r-1} P_{r,m_r-1} + \pi_{r,m_r} (P_r - P_{r,m_r-1}) \leq s_r \end{cases}, \quad r = 1, \dots, N_r \quad (6-d)$$

$$\lambda = \arg \min_{P_i} \sum_{i=1}^{N_g} (a_i P_i^2 + b_i P_i + c_i) \quad (6-e)$$

$$s.t. \quad \sum_{i=1}^{N_g} P_i = D: \quad \lambda \quad (6-f)$$

$$P_i^{\min} \leq P_i \leq P_i^{\max}, \quad i = 1, \dots, N_g \quad (6-g)$$

Hence, (6) is a bi-level model. Traditionally, the combinatorial nature of bi-level programming can be observed by studying the single-level reformulation which is obtained by replacing the inner level problem with its KKT (Karush-Kuhn-Tucker) condition. Furthermore, it leads to a mixed integer programming with big M approach and dummy logic variables. However, only if both the inner

and outer model are convex, KKT condition can be a necessary and sufficient optimality condition. Here, the inner model of (6) is convex, but the outer model is nonconvex due to the existence of a bilinear term λD . Instead of using KKT condition, in order to solve this issue, the method to obtain the closed form of the inner model and further transform the bi-level model into a single level model is proposed in Section. III. It need be noted that this bilinear term λD is special, where λ is uniquely determined by the inner model.

3. A Method to Closed Form of Inner Model

The inner model is a special quadratic programming model which contains only one equality and bound constraints for each variables, such that

$$\min_{P_1, \dots, P_{N_g}} \sum_{i=1}^{N_g} F_i(P_i) \quad (7-a)$$

$$s.t. \quad \sum_{i=1}^{N_g} P_i = D: \quad \lambda \quad (7-b)$$

$$P_i^{\min} \leq P_i \leq P_i^{\max}, \quad i = 1, \dots, N_g \quad (7-c)$$

Without considering the bound constraints (7-c), the Lagrangian function is expressed as

$$L(P_i) = \sum_{i=1}^{N_g} (a_i P_i^2 + b_i P_i + c_i) + \lambda \left(D - \sum_{i=1}^{N_g} P_i \right) \quad (8)$$

According to KKT condition, it yields

$$\begin{cases} \lambda = \left(D + \sum_{i=1}^{N_g} \frac{b_i}{2a_i} \right) / \sum_{i=1}^{N_g} \frac{1}{2a_i} \\ P_i = \frac{\lambda - b_i}{2a_i} \quad i = 1, \dots, N_g \end{cases} \quad (9)$$

It can be found that the solution and multiplier are the linear increasing functions of D , such as $\lambda(D)$ and $P_i(D)$. Considering capacity limits $P_i^{\min} \leq P_i \leq P_i^{\max}$ of $P_i(D)$, $\lambda(D)$ and $P_i(D)$ become piecewise linear functions when different upper and lower bound constraints are active [29].

In order to obtain the closed form of $\lambda(D)$, the space $\Gamma = (A, \{P^{\min}, P^{\max}\})$ is defined, where A is the set of generators. Since $P_i(D)$ and $\lambda(D)$ are piecewise linear functions, in each interval $D_n \leq D \leq D_{n+1}$, the units can be partitioned into three groups: Ω_n , Θ_n and Ξ_n , where Ω_n denotes the units with minimum capacity limits active; Θ_n denotes the marginal units; and Ξ_n denotes the units with maximum capacity limits active. The closed forms of $P_i(D)$ and $\lambda(D)$ are:

$$P_i(D) = \begin{cases} y & i \in \Theta_n \\ P_i^{\min} & i \in \Omega_n \\ P_i^{\max} & i \in \Xi_n \end{cases}, \quad y = \frac{D - \sum_{k \in \Omega_n} P_k^{\min} - \sum_{k \in \Xi_n} P_k^{\max} + \sum_{k \in \Theta_n} \frac{b_k}{2a_k}}{a_i \sum_{k \in \Theta_n} \frac{1}{a_k}} \quad (10)$$

$$\lambda(D) = \frac{2D}{\sum_{k \in \Theta_n} \frac{1}{a_k}} + \frac{1}{\sum_{k \in \Theta_n} \frac{1}{a_k}} \sum_{k \in \Theta_n} \frac{b_k}{a_k} - \frac{2}{\sum_{k \in \Theta_n} \frac{1}{a_k}} \left(\sum_{k \in \Omega_n} P_k^{\min} + \sum_{k \in \Xi_n} P_k^{\max} \right) \quad (11)$$

The Ω_n , Θ_n and Ξ_n as well as the segment D_n should be determined to formulate the closed form of the two piecewise linear function $P_i(D)$ and $\lambda(D)$.

Now, define by g the isomorphism

$$V(i, x) = 2a_i x + b_i : (A, \{P^{\min}, P^{\max}\}) \rightarrow \mathbb{R} \quad (12)$$

Obviously, the cardinality of $\{V(i, x)\}$ is $2N_g$. Furthermore, set an increasing order for $V(\Gamma)$, such that

$$V(s_1, x_1) < V(s_2, x_2) < \dots < V(s_{2N_g}, x_{2N_g}) \quad (13)$$

where (s_i, x_i) belongs to Γ .

In addition, if the model (7) is feasible, the load demand satisfies $\sum_{i=1}^{N_g} P_i^{\min} \leq D \leq \sum_{i=1}^{N_g} P_i^{\max}$.

when $D = \sum_{i=1}^{N_g} P_i^{\min}$, $\Omega_0 = \{1, 2, \dots, N_g\}$, $\Theta_0 = \emptyset$ and $\Xi_0 = \emptyset$;

when $D = \sum_{i=1}^{N_g} P_i^{\max}$, $\Omega_{2N_g} = \emptyset$, $\Theta_{2N_g} = \emptyset$ and $\Xi_{2N_g} = \{1, 2, \dots, N_g\}$.

Furthermore, according to the order (13), the three sequence can be updated by

If $x_i = P_i^{\max}$: $\Omega_n = \Omega_{n-1}$, $\Theta_n = \Theta_{n-1} - \{i\}$, $\Xi_n = \Xi_{n-1} \cup \{i\}$;

If $x_i = P_i^{\min}$: $\Omega_n = \Omega_{n-1} - \{i\}$, $\Theta_n = \Theta_{n-1} \cup \{i\}$, $\Xi_n = \Xi_{n-1}$.

In this term, there are $2N_g + 1$ states, which includes 2 points $\{\Omega_0, \Theta_0, \Xi_0\}$ and $\{\Omega_{2N_g}, \Theta_{2N_g}, \Xi_{2N_g}\}$, and $2N_g - 1$ piecewise linear functions. Meanwhile, it should be noted that the number of piecewise linear functions is not necessarily $2N_g - 1$. For the following cases, the number can be smaller.

(i) If there exists $V(s_i, x_i) = V(s_j, x_j)$, the three sequence should be updated at the same time and the number of states will be one less.

(ii) If $\Theta_n = \emptyset$, the piecewise linear function from $\{\Omega_n, \Theta_n, \Xi_n\}$ is actually a point, which leads to $\Theta_n = \Theta_{n-1}$.

Now, assume there are $m+1$ segments and m piecewise linear functions, then the segments can be obtained by

$$D_n = \begin{cases} \frac{1}{2} \sum_{k \in \Theta_n} \frac{2a_{s_n} P_{s_n}^{\min} + b_{s_n} - b_k}{a_k} + \sum_{i \in \Omega_n} P_i^{\min} + \sum_{j \in \Xi_n} P_j^{\max} & \text{if } x_{s_n} = P_{s_n}^{\min} \\ \frac{1}{2} \sum_{k \in \Theta_n} \frac{2a_{s_n} P_{s_n}^{\max} + b_{s_n} - b_k}{a_k} + \sum_{i \in \Omega_n} P_i^{\min} + \sum_{j \in \Xi_n} P_j^{\max} & \text{if } x_{s_n} = P_{s_n}^{\max} \end{cases} \quad (14)$$

Furthermore, in each interval $D_n < D < D_{n+1}$, the linear function of $\lambda(D)$ can be determined by (11), according to the groups $\{\Omega_n, \Theta_n, \Xi_n\}$ by the above discussion.

4. Mixed Integer Conic Programming for the Proposed Model with Demand Response Bids

After obtaining the segments, the closed form of λ with respect to D can be written as (15), where the linear function in each interval can be computed by (11), leading to

$$\lambda = \begin{cases} h_1 D + g_1 & \text{if } D_0 \leq D \leq D_1 \\ h_2 D + g_2 & \text{if } D_1 \leq D \leq D_2 \\ \vdots & \dots \\ h_m D + g_m & \text{if } D_{m-1} \leq D \leq D_m \end{cases} \quad (15)$$

where h_i is positive within each sub-region due to the decreasing property of multiplier. However, it is still difficult to ensure the convexity of λ in the whole space.

At first, the bi-linear term in objective function (6-a) to maximize $-\lambda D$ can be formulated as

$$\lambda D = \begin{cases} h_1 D^2 + g_1 D & \text{if } D_0 \leq D \leq D_1 \\ h_2 D^2 + g_2 D & \text{if } D_1 \leq D \leq D_2 \\ \vdots & \dots \\ h_m D^2 + g_m D & \text{if } D_{m-1} \leq D \leq D_m \end{cases} \quad (16)$$

Since D is a non-negative value, additional binary variables y_i and continuous variables Z_i can be employed to simplify (6-a) into (17), where $i=1, \dots, m$.

$$\lambda D = \max \{h_1 Z_1^2 + g_1 Z_1, h_2 Z_2^2 + g_2 Z_2, \dots, h_m Z_m^2 + g_m Z_m\} \quad (17-a)$$

with

$$\sum_{i=1}^m y_i = 1, \quad D = \sum_{i=1}^m Z_i, \quad D_{i-1} y_i \leq Z_i \leq D_i y_i, \quad y_i = \{0, 1\}, \quad i=1, \dots, m \quad (17-b)$$

Take (17) into (6-a) with introducing one dummy variable t , hence

$$Z = \max \quad \lambda_0 D - t - \sum_{r=1}^{N_r} s_r \quad (18-a)$$

with additional constraints

$$\begin{cases} h_1 Z_1^2 + g_1 Z_1 \leq t \\ h_2 Z_2^2 + g_2 Z_2 \leq t \\ \vdots \\ h_m Z_m^2 + g_m Z_m \leq t \end{cases} \quad (18-b)$$

For each constraint of (18-b), it can be reformulated as

$$h_i Z_i^2 + g_i Z_i \leq t \Leftrightarrow h_i Z_i^2 + g_i Z_i - t \leq 0$$

$$\Leftrightarrow \left\| \frac{(1 + g_i Z_i - t)}{2} \right\|_2 \leq \frac{(1 - g_i Z_i + t)}{2}, \quad i=1, \dots, m \quad (19)$$

Equation (19) is a second order cone constraints. Take (19) and (13) into (DR1), (6) can be transformed into a mixed integer conic programming model, such that

$$(\text{DR2}) \quad Z = \max \quad \lambda_0 D - t - \sum_{r=1}^{N_r} s_r \quad (20-a)$$

$$s.t. \quad 0 \leq D_r \leq D_r^{\max}, \quad r=1, \dots, N_r \quad (20-b)$$

$$D = \bar{D} - \sum_{r=1}^{N_r} D_r \quad (20-c)$$

$$\begin{cases} \pi_{r,1} P_r \leq s_r \\ \pi_{r,1} P_{r,1} + \pi_{r,2} (P_r - P_{r,1}) \leq s_r \\ \dots \\ \pi_{r,m_r-1} P_{r,m_r-1} + \pi_{r,m_r} (P_r - P_{r,m_r-1}) \leq s_r \end{cases}, \quad r=1, \dots, N_r \quad (20-d)$$

$$\left\| \frac{(1 + g_i Z_i - t)}{2} \right\|_2 \leq \frac{(1 - g_i Z_i + t)}{2}, \quad i=1, \dots, m \quad (20-e)$$

$$\sum_{i=1}^n y_i = 1, \quad D = \sum_{i=1}^n Z_i, \quad D_{i-1} y_i \leq Z_i \leq D_i y_i$$

$$y_i = \{0, 1\}, \quad i=1, \dots, m \quad (20-f)$$

Relaxing the binary variables into continuous ones, the above model becomes a conic programming which is convex. As a result, (16) is essentially a mixed integer convex programming method that be efficiently solved by branch-and-cut method to achieve the global optimum using commercial solvers, such as CPLEX, MOSEK, and GUROBI.

Especially, if (15) is a convex piecewise linear function, such that $h_1 \leq h_2 \leq \dots \leq h_m$, the integer variables in (20) can be eliminated, which leads to

$$(\text{DR3}) \quad Z = \max \quad \lambda_0 D - t - \sum_{r=1}^{N_r} s_r \quad (21-a)$$

$$s.t. \quad 0 \leq D_r \leq D_r^{\max}, \quad r=1, \dots, N_r \quad (21-b)$$

$$D = \bar{D} - \sum_{r=1}^{N_r} D_r \quad (21-c)$$

$$\begin{cases} \pi_{r,1} P_r \leq s_r \\ \pi_{r,1} P_{r,1} + \pi_{r,2} (P_r - P_{r,1}) \leq s_r \\ \dots \\ \pi_{r,m_r-1} P_{r,m_r-1} + \pi_{r,m_r} (P_r - P_{r,m_r-1}) \leq s_r \end{cases}, \quad r=1, \dots, N_r \quad (21-d)$$

$$\left\| \frac{(1 + g_i D - t)}{2} \right\|_2 \leq \frac{(1 - g_i D + t)}{2}, \quad i=1, \dots, m \quad (21-e)$$

4. Numerical Results

A. Test on the closed form of $\lambda(D)$

The method of obtaining the closed form of $\lambda(D)$ is performed and further verified in this subsection based on a 9-bus system with 3 generation units. The parameters of the generation units are available as in Table I. The steps of testing are presented as follows:

Step 1: Compute $V(i, x) = 2a_i x + b_i$ as

$$V(1, P_1^{\min}) = 7.2; V(2, P_2^{\min}) = 2.9; V(3, P_3^{\min}) = 3.45;$$

$$V(1, P_1^{\max}) = 60; V(2, P_2^{\max}) = 52.2; V(3, P_3^{\max}) = 67.15;$$

Step 2: Order $V(i, x)$ in an increase sequence, such that

Therefore, if $\left[\bar{D} - \sum_{r=1}^{N_r} D_r, \bar{D}\right] \subseteq [70.60, 820]$, (21) can be

Step 3: The sequence groups can be formulated as

$$\begin{aligned} \Omega_0 &= \{1, 2, 3\}, \Theta_0 = \emptyset, \Xi_0 = \emptyset; & \Omega_1 &= \{1, 3\}, \Theta_1 = \{2\}, \Xi_1 = \emptyset; \\ \Omega_2 &= \{1\}, \Theta_2 = \{2, 3\}, \Xi_2 = \emptyset; & \Omega_3 &= \emptyset, \Theta_3 = \{1, 2, 3\}, \Xi_3 = \emptyset; \\ \Omega_4 &= \emptyset, \Theta_4 = \{1, 3\}, \Xi_4 = \{2\}; & \Omega_5 &= \emptyset, \Theta_5 = \{3\}, \Xi_5 = \{1, 2\}; \\ \Omega_6 &= \emptyset, \Theta_6 = \emptyset, \Xi_6 = \{1, 2, 3\}; \end{aligned}$$

Step 4: Compute the break points by (14)

$$D_1 = 30; D_2 = 33.24; D_3 = 70.60; D_4 = 723.53; D_5 = 790.82; D_6 = 820;$$

Step 5: Compute each piecewise linear function by (11)

For $D_1 \leq D \leq D_2$, it leads to $\Omega_1 = \{1, 3\}$, $\Theta_1 = \{2\}$, $\Xi_1 = \emptyset$, and we have $\lambda(D) = 0.1700D - 2.2000$;

For $D_2 \leq D \leq D_3$, it leads to $\Omega_2 = \{1\}$, $\Theta_2 = \{2, 3\}$, $\Xi_2 = \emptyset$, and we have $\lambda(D) = 0.1004D + 0.1145$;

For $D_3 \leq D \leq D_4$, it leads to $\Omega_3 = \emptyset$, $\Theta_3 = \{1, 2, 3\}$, $\Xi_3 = \emptyset$, and we have $\lambda(D) = 0.0689D + 2.3342$;

For $D_4 \leq D \leq D_5$, it leads to $\Omega_4 = \emptyset$, $\Theta_4 = \{1, 3\}$, $\Xi_4 = \{2\}$, and we have $\lambda(D) = 0.1159D - 31.6667$;

For $D_5 \leq D \leq D_6$, it leads to $\Omega_5 = \emptyset$, $\Theta_5 = \{3\}$, $\Xi_5 = \{1, 2\}$, and we have $\lambda(D) = 0.2450D - 133.7500$;

Hence, the closed form of $\lambda(D)$ is expressed as

$$\lambda(D) = \begin{cases} 0.1700D - 2.2000 & 30 \leq D \leq 33.24 \\ 0.1004D + 0.1145 & 33.24 \leq D \leq 70.60 \\ 0.0689D + 2.3342 & 70.60 \leq D \leq 723.53 \\ 0.1159D - 31.6667 & 723.53 \leq D \leq 790.82 \\ 0.2450D - 133.7500 & 790.82 \leq D \leq 820 \end{cases} \quad (22)$$

TABLE I. PARAMETER OF 9 BUS SYSTEM

Units	a_i (\$/MW ²)	b_i (\$/MW)	c_i (\$)	P^{\min} (MW)	P^{\max} (MW)
1	0.1100	5.0	150	10	250
2	0.0850	1.2	600	10	300
3	0.1225	1.0	335	10	270

In order to verify the proposed method, the shadow prices with different load demands are computed by the economic dispatch model (7) by MOSEK, where the tested load demands are set from 30 to 820 MW with 40 MW as step length. The shadow price under same load demand can be obtained by the closed form (22). Moreover, the results of ED and closed form are compared in Fig. 3 to depict that the closed form of shadow price with respect to the total load demand is as the same as that from performing ED. Therefore, this closed form has been verified to be able to simplify the inner model of the original bi-level programming problem. Especially, it can be found from the closed form (22) that this piecewise linear function is neither complete convex nor complete concave: The function is concave with $30 \leq D \leq 70.60$ and convex with $70.60 \leq D \leq 820$. Recall model (20) and (21),

the range of $D \left[\bar{D} - \sum_{r=1}^{N_r} D_r, \bar{D} \right]$ can be easily obtained.

applied for the proposed demand response bidding model; otherwise, (20) can be utilized for which need to introduce additional integer variables.

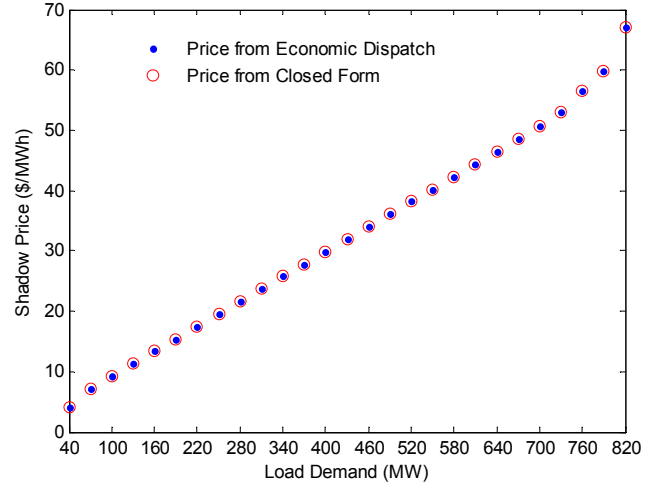


Fig.3 Comparison of the closed form and the economic dispatch

B. Test on the proposed bi-level programming model

IEEE 118-bus test system has been studied to validate the effectiveness of the proposed bi-level programming model on providing optimal operating strategy for LSEs with considering demand response. For this case study, the generator parameters are from MATPOWER [30], where there are 19 units in service. According to the economic dispatch model (7), the total load demand is set as 5500 MW and the real-time price without DR is 46.0435 \$/MWh. The proposed approach is performed using MATLAB, and MOSEK on a personal computer with Intel® Core™ i5 Duo Processor T420 (2.50 GHz) and 4 GB RAM.

In this case study, there are three consumers participating the DR, whose bidding curves are given in Fig. 4. The maximum reducible demand for each consumer is 400 MW. Hence, the practical load with the consideration of DR should be within the range of [4300, 5500] MW. In addition, the closed form of shadow price λ by (11) has been obtained by the proposed method as (23). Note, there are eighteen pieces in total and only seven pieces are presented here due to space limitation. Since (23) is convex, the model (21) can be applied to solve this bi-level programming model.

$$\lambda(D) = \begin{cases} 0.0046D + 19.8757 & 0 \leq D \leq 5098.6 \\ 0.0053D + 16.2497 & 5098.6 \leq D \leq 5267.9 \\ 0.0061D + 12.2026 & 5267.9 \leq D \leq 5309.3 \\ 0.0070D + 7.1000 & 5309.3 \leq D \leq 5402.8 \\ 0.0082D + 1.0231 & 5402.8 \leq D \leq 5404.4 \\ 0.0097D - 7.3442 & 5404.4 \leq D \leq 5533.6 \\ 0.01145D - 17.0018 & 5533.6 \leq D \leq 5670.42 \end{cases} \quad (23)$$

Taking different retail prices into consideration for illustration, real-time prices in ISO/RTOs' market with and

without DR are shown in Fig. 5 for comparison. The results show the real-time price with DR is relatively lower. Although the real-time price λ is reduced by shedding demand, LSEs still need to compensate the consumers' based on their bids in DRs according to. Therefore, the proposed bi-level programming model can be applied to optimize the value of the DR.

Fig. 6 depicts the optimal value of load shedding in DRs solved by the proposed method. Generally, when the retail price is relatively lower to the real-time price, in order to avoid losses, LSE will have the incentive to perform DR to reduce the real-time price. When the retail price is relatively higher than the real-time price, LSE will not have the incentive to perform DR.

The profit of LSE is plotted in Fig. 7 to demonstrate that the LSE's profit with DR can be increased comparing to it without DR. Specifically, the profit of LSE consists of two parts: (i) the cost of compensating consumers in DR; (ii) the profit between purchasing electricity from ISO/RTOs and selling it to consumers. With the increment of retail price, the cost of DR will be reduced until LSE can meet the demand by directly buying from ISO/RTO without DR. Meanwhile, since the real-time price will rise if buying more electricity from ISO/RTO, DR also can help further reduce the real-time price for LSEs to maximize their profit even when the retail price is high.

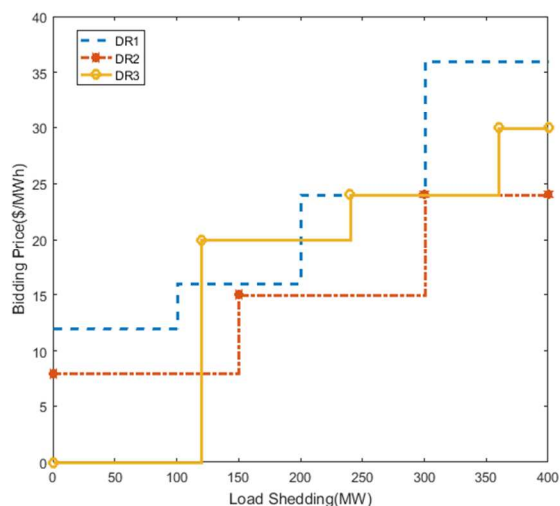


Fig. 4. Bidding curves of three DRs

It should be noted that in practical electricity market, the retail price is determined before LSE makes decision for DR bidding. For example, if the retail price is 40 \$/MWh, the optimal load shedding strategy for the three DR participants should be $D_{r,1}=200$ MW, $D_{r,2}=300$ MW, $D_{r,3}=240$ MW; if the retail price is 60 \$/MWh, the optimal load shedding strategy for the three DR participants should be $D_{r,1}=92.74$ MW, $D_{r,2}=150$ MW, $D_{r,3}=0$ MW. For the proposed bidding model, LSE will need to forecast the total load demand \bar{D} , which is usually uncertain and need to be addressed. Therefore, different realizations of load demand within [5000, 5600] MW have been considered. The results of load shedding for DR with retail price being

40 \$/MWh and 60 \$/MWh are presented in Fig. 8 and Fig. 9, respectively. From the simulation results, it is obvious that, with increasing the load demand, more load shedding for DR is utilized. According to the result, there is a fact that the load shedding of DRs may be constant within certain interval. For Fig. 8, the results are constant when $\bar{D}=[5200,5540]$ MW; for Fig. 9, the results are constant when $\bar{D}=[5300,5400]$ MW; and, both of them are constant near 5500 MW.

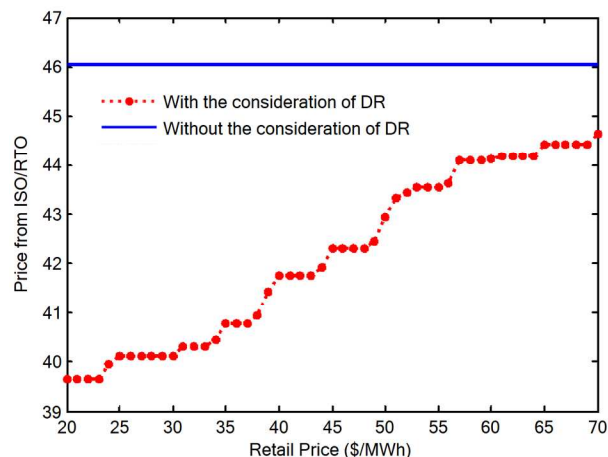


Fig. 5. Price from ISO/RTO with different retail price

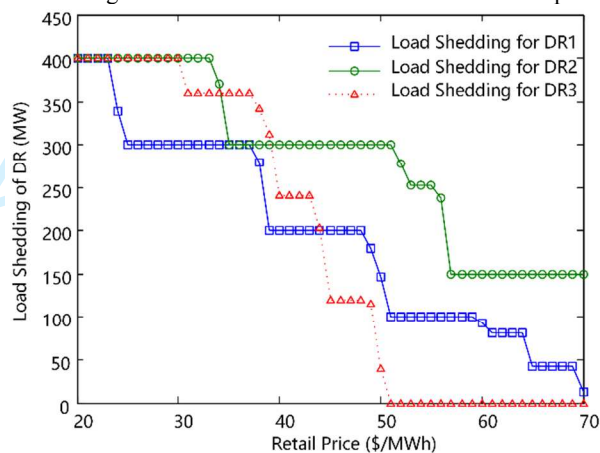


Fig. 6. Bidding results of load shedding of each DR by LSE

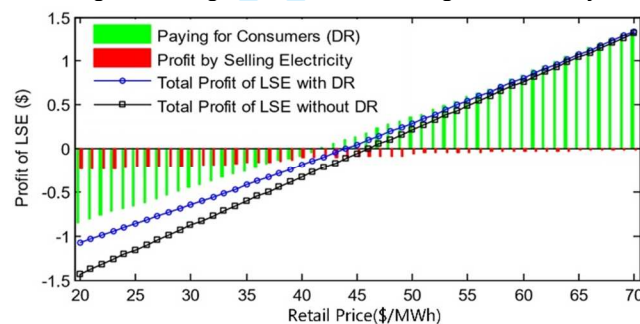


Fig. 7. Comparison of profit of LSE with and without DR

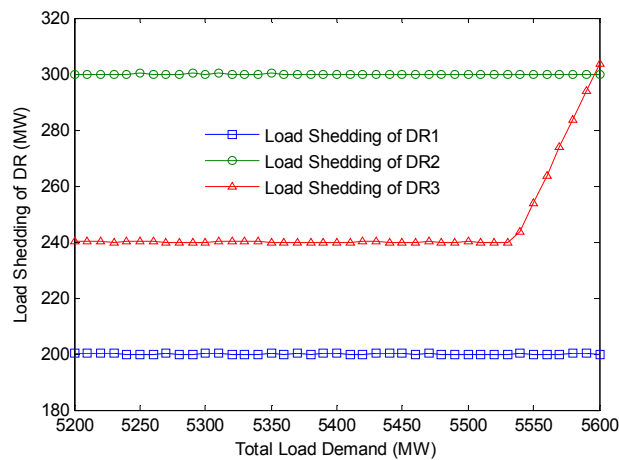


Fig. 8. Load Shedding of DR under retail price 40 \$/MWh

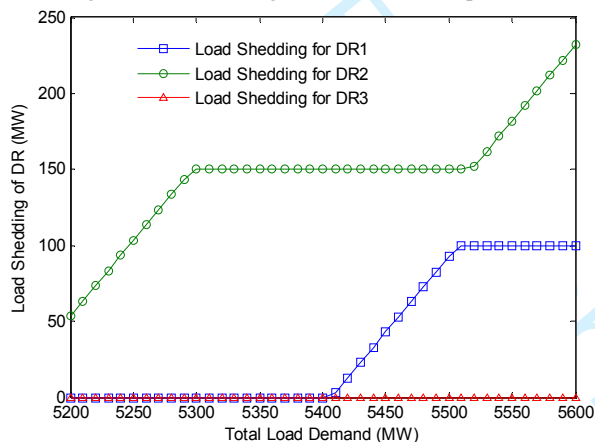


Fig. 9. Load Shedding of DR under retail price 60 \$/MWh

When the consumers' bidding price is set to be as five times as the value in Fig. 4, and the real-time price with DR under different retail prices and the optimized load shedding of each DR can be observed in Fig. 10 and Fig. 11 respectively. Compared with Fig. 5 and Fig. 6, it is similar that the lower retail price is, the larger amount of load shedding is, and the lower real-time price will be. Meanwhile, due to the high bidding price, the amount of load shedding in Fig. 11 is less than that Fig. 6. Also, due to the high bidding price, when the retail price is higher than 55 \$/MWh, performing DR will not further increase LSEs profit but only bring them extra costs in compensating consumers in DR. Fig. 12 is the comparison of LSEs' profit with and without DR. The results show that DR only slightly improves the LSEs' profit. Comparing with Fig. 7, it concludes that 1) LSEs can benefit from the DRs, and 2) lower bidding price is, the more profit growth can be achieved. Finally, the results of load shedding for DR with retail price being 40 \$/MWh and 60 \$/MWh are presented in Fig. 13 and Fig. 14, respectively. Compared with those in Fig. 8 and Fig. 9, it can be observed that, with increasing the load demand, only load shedding for DR2 is utilized and for the same retail price, there needs less load shedding, because the DR bidding price is higher than before (five times as the value in Fig. 4).

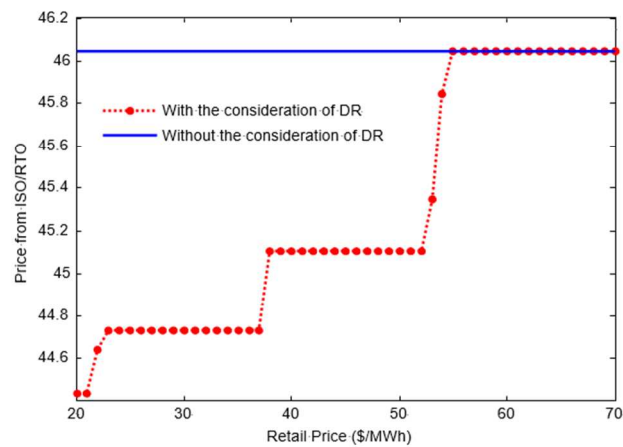


Fig. 10. Price from ISO/RTO with different retail price

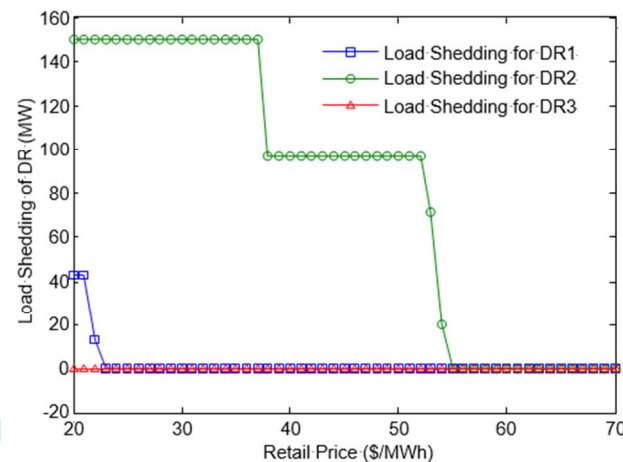


Fig. 11. Bidding results of load shedding of each DR by LSE

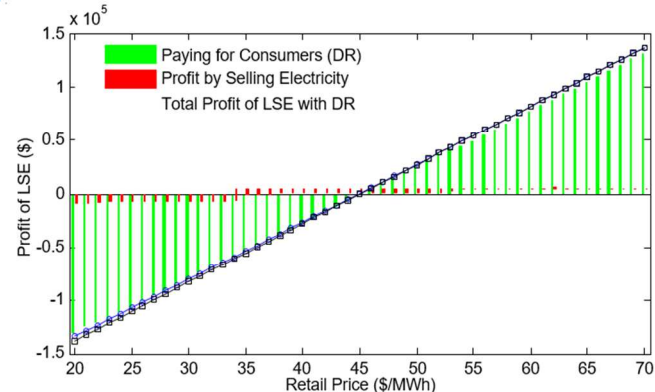


Fig. 12. Comparison of profit of LSE with and without DR

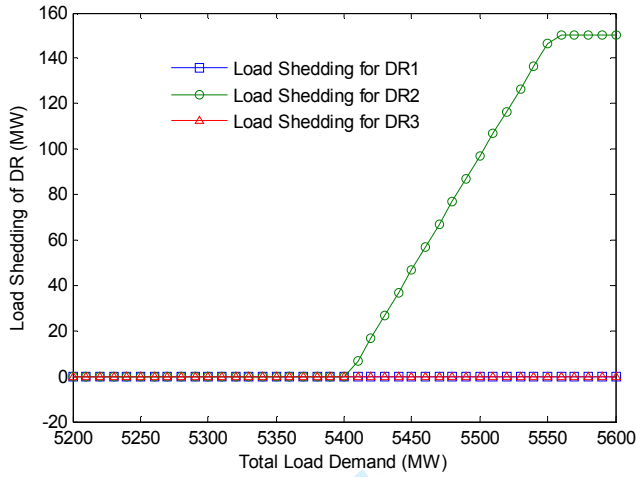


Fig. 13. Load Shedding of DR under retail price 40 \$/MWh

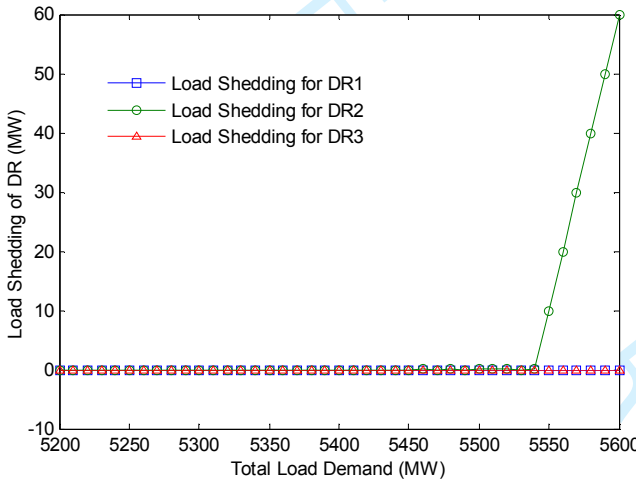


Fig. 14. Load Shedding of DR under retail price 60 \$/MWh

5. Conclusions

This paper proposed a bi-level optimization model with the consideration of demand response bidding to maximize the total profit of LSEs. The contributions of our work are as follows:

The consumers participate DR through setting their own bidding prices. Therefore, LSEs are able to determine the optimal reward value of DR as well as the amount of demanded electricity without knowing the accurate demand response characteristics.

An original method has been implemented to solve the bi-level optimization model. The closed form of shadow price function with respect to the total load demand is derived, which can greatly reduce the complexity of the proposed bi-level programming model and leads to a mixed integer second order cone programming to achieve the global optimality. Most importantly, the proposed approach to the closed form of shadow price can also be applied to other bi-level programming models.

Finally, it should be noted that we consider the optimal strategy as a real-time problem in this paper. Therefore, only a single-period economic dispatch model is considered. However, considering the coupling of

different time periods, a multi-period dynamic economic dispatch model will add another dimension into the proposed model, which will be more complex and we may research in future works.

Acknowledgement

This work was supported in part by National Key Research and Development Program of China (2016YFB0901100), in part by National Natural Science Foundation of China (Grant 51607137), in part by China Postdoctoral Science Foundation (2017T100748), in part by Natural Science Basis Research Plan in Shaanxi Province of China (2016JQ5015) and in part by the project of State Key Laboratory of Electrical Insulation and Power Equipment in Xi'an Jiaotong University (EIPE17205, EIPE16301).

References

[1]. Rahimi, F., and Ipakchi, A., "Demand Response as a Market Resource Under the Smart Grid Paradigm," *IEEE Trans. Smart Grid*, vol. 1, no. 1, pp. 82-88, 2010.

[2]. Nguyen, D. T., Negnevitsky, M., and Groot, M., de, "Pool-Based Demand Response Exchange - Concept and Modeling," *IEEE Trans. Power Syst.*, vol. 26, no. 3, pp. 1677-1685, 2011.

[3]. Samarakoon, K., Ekanayake, J., and Jenkins, N., "Reporting Available Demand Response," *IEEE Trans. SG*, vol. 4, no. 4, pp. 1842-1851, 2013.

[4]. Albadi, M., H., and El-Saadany, E., F., "Demand Response in Electricity Markets: An Overview," *IEEE Power Engineering Society General Meeting*, pp. 1-5, 2007.

[5]. Cappers, P., Goldman, C., and Kathan, D., "Demand response in U.S. electricity markets: Empirical evidence," *Energy*, vol. 35, no. 4, pp. 1526-1535, 2010.

[6]. Chen, C., Wang, J., and Kishore, S., "A Distributed Direct Load Control Approach for Large-Scale Residential Demand Response," *IEEE Trans. Power Syst.*, vol. 29, no. 5, pp. 2219-2228, 2014.

[7]. Chen, C., S., and Leu, J., T., "Interruptible load control for Taiwan Power Company," *IEEE Trans. Power Syst.*, vol. 5, no. 2, pp. 460-465, 1990.

[8]. Wang, Y., Pordanjani, I., R., and Xu, W., "An Event-Driven Demand Response Scheme for Power System Security Enhancement," *IEEE Trans. Smart Grid*, vol. 2, no. 1, pp. 23-29, 2011.

[9]. Nguyen, D., T., Negnevitsky, M., and Groot, M., de, "Market-Based Demand Response Scheduling in a Deregulated Environment," *IEEE Trans. Smart Grid*, vol. 4, no. 4, pp. 1948-1956, 2013.

[10]. Philpott, B., and Pettersen, E., "Optimizing demand-side bids in day-ahead electricity markets," *IEEE TPWRS*, vol. 21, no. 2, pp. 488-498, 2006.

[11]. Duy, N., Negnevitsky, Thanh, M., and Groot, M., de, "Pool-Based Demand Response Exchange - Concept and Modeling," *IEEE Trans. Power Syst.*, vol. 26, no. 3, pp. 1677-1685, 2011.

[12]. Vasileva, E., and Viljainen, S., "Capacity market as an incentive for demand response in Russia," *11th Int. Conf. European Energy Market (EEM)*, pp. 1-5, 2014.

[13]. Ma, O., Alkadi, N., Cappers, P., Denholm, P., et al., "Demand Response for Ancillary Services," *IEEE Trans. Smart Grid*, vol. 4, no. 4, pp. 1988-1995, 2013.

[14]. Andruszkiewicz, J., Lorenc, J., and Michalski, A., "Efficiency improving of time of use tariffs for residential customer to create more valuable demand

- response resource," *11th Int. Conf. European Energy Market (EEM)*, pp. 1-5, 2014.
- [15]. Aghaei, J., and Alizadeh, M., I., "Critical peak pricing with load control demand response program in unit commitment problem," *IET Gen., Trans. & Dist.*, vol. 7, no. 7, pp. 681-690, 2013.
- [16]. Nagarajan, K., V., "Benefit-Cost Methodology for Evaluating Peak-Load Pricing System Implementation for Electricity Rates," *IEEE Trans. Syst., Man and Cybern.*, vol. 10, no. 9, pp. 541-547, 1980.
- [17]. Roozbehani, M., Dahleh, M. A., and Mitter, S. K., "Volatility of Power Grids Under Real-Time Pricing," *IEEE Trans. Power Syst.*, vol. 27, no. 4, pp. 1926-1940, 2012.
- [18]. Jonghe, C. De, Hobbs, B. F., and Belmans, R., "Optimal generation mix with short-term demand response and wind penetration," *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 830-839, 2012.
- [19]. Shuhui, L., Dong, Z., Roget, A. B., and O'Neill, Z., "Integrating Home Energy Simulation and Dynamic Electricity Price for Demand Response Study," *IEEE Trans. Smart Grid*, vol. 5, no. 2, pp. 779-788, 2014.
- [20]. Vlachos, A. G., and Biskas, P. N., "Demand Response in a Real-Time Balancing Market Clearing With Pay-As-Bid Pricing," *IEEE Trans. Smart Grid*, vol. 4, no. 4, pp. 1966-1975, 2013.
- [21]. Fang, X., Hu, Q., Li, F. and et al, "Coupon-Based Demand Response Considering Wind Power Uncertainty: A Strategic Bidding Model for Load Serving Entities," *IEEE Trans. Power Syst.*, in-press, 2015.
- [22]. Zhao, C., Wang, J., Watson, J. P., and Guan, Y., "Multi-Stage Robust Unit Commitment Considering Wind and Demand Response Uncertainties," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2708-2717, 2013.
- [23]. Taylor, J. A., and Mathieu, J. L., "Index Policies for Demand Response," *IEEE Trans. Power Syst.*, vol. 29, no. 3, pp. 1287-1295, 2014.
- [24]. Niu, W., and Li, Y., "Uncertain optimization decision of interruptible load in Demand Response program," *IEEE Innovative Smart Grid Technologies - Asia*, pp. 675-679, 2014.
- [25]. Wang, Q., Wang, J., and Guan, Y., "Stochastic Unit Commitment With Uncertain Demand Response," *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 562-563, 2013.
- [26]. Xu, Y., Xie, L., and Singh, C., "Optimal scheduling and operation of load aggregators with electric energy storage facing price and demand uncertainties," *North American Power Symposium* pp. 1-7, 2011.
- [27]. Wu, D., Aliprantis, D. C., and Ying, L., "Load Scheduling and Dispatch for Aggregators of Plug-In Electric Vehicles," *IEEE Trans. Smart Grid*, vol. 3, no. 1, pp. 368-376, 2012.
- [28]. Zhang, W., Lian, J., Chang, C.-Y., and Kalsi, K., "Aggregated Modeling and Control of Air Conditioning Loads for Demand Response," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4655-4664, 2013.
- [29]. Zimmerman, R. D., Murillo-Sánchez, C. E., and Thomas, R. J., "MATPOWER: Steady-State Operations, Planning, and Analysis Tools for Power Systems Research and Education," *IEEE Trans. Power Syst.*, vol. 26, no. 1, pp. 12-19, Feb. 2011.
- [30]. Bay, L., Grau, J., Ruiz, M., Su, P., "An analytic solution for some separable convex quadratic programming problems with equality and inequality constraints," *J. Math. Inequalities*, vol. 4, no. 3, pp. 453-465, 2010.